CUBES ETC

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Contents

1 Introduction

Here's a brief overview of the code and formulae that are used in the code of the Exposure Time Calculator (ETC) for CUBES. The code provides for a point source the signal-to-noise ratio (SNR) for pixel in a given exposure time (or vice-versa) at a given wavelength. Moreover, the ETC can compute the liming magnitude (in AB unit) achievable at a given wavelength, in a given exposure time and at a given SNR.

The magnitude are in AB units.

Calculations are performed by a Python script that is called by a web application written in Java and JavaScript.

A preliminary version will be soon available at `http://archives.ia2.inaf.it/cubes/` for internal use only.

2 Input Parameters of CUBES ETC

All inputs are divided in User Inputs and Cubes Parameters. Every parameter is imported directly to the script after its check in the corresponding acceptable range. The following Table 1 shows the input parameters and their acceptable ranges.

Table 1: User Input Parameters

Default Value	Script Variable Name	Acceptable Value Range
ExpT	`SNRorExpT`	`'ExpT' or 'SNR'`
	`Spectrum`	`Synthetic stellar spectra, Quasar and Custom spectrum`
12	`V`	[-2:30]
3500	`wl`	[2900 - 4100]
0	`z`	[0:7]
22	`Vsky`	[18 - 24]
0.7	`seeing`	[0.0:20]
1.0	`airmass`	[1.0:3.0]
1	`nbinx`	[1:10]
1	`nbiny`	[1:10]
	`ExpT` or `SNR`	≥ 0

where:

`SNRorEXPT` is a variable that allows you to choose if you want to calculate SNR given the exposure time (ExpT) or vice versa; default value is ExpT;

`Spectrum` is the input spectrum in photon $\mathrm{cm^{-2}\,s^{-1}}$ Å^{-1} at a magnitude V_0;

`V` is V magnitude of the target;

`wl` is the wavelength in Å at which SNR (or ExpT) is calculated;

`z` is redshift of the quasar;

`Vsky` is the V sky brightness in mag arcsec^{-2};

`seeing` is seeing Γ_0 at 5500 Å in arcsec;

`airmass` is the airmass X;

`nbinx` is the CCD binning in the dispersion direction (X);

`nbiny` is is the CCD binning in the spatial direction (Y);

`ExpT` is the input exposure time in sec (or SNR value);

3 Hidden Parameters: CUBES set-up

Table 2: CUBES set-up

Default value	Script Variable Name	Acceptable Value Range
0.25	`slice_width`	0.01 - 2.1
10.0	`slice_height`	0.01 - 15
1.5	`slit_width`	1.2 - 1.6
10.0	`slit_height`	0.01 - 15
6	`n_slices`	4 - 8
3369	`sig_ch1`	3339 - 3399
3086	`sig_ch2`	3056 - 4016
2834	`sig_ch3`	2834 - 2874
32.8	`alp`	30 - 45
3094	`fcoll`	3084 - 3104
20.63	`fno_coll`	20.0 - 21.0
464	`fcam`	450 - 470
3.09	`fno_cam`	3 - 4
1	`m`	1
3218	`wlc_ch1`	3208- 3228
3513	`wlc_ch2`	3503- 3523
3825	`wlc_ch3`	3815- 3835
0.0005	`DC`	
2.5	`ron`	
15	`pixsize`	
4096	`NXpix`	
4096	`NYpix`	
8200	`d_tel`	
0.14	`eps`	

where:

`slice_width` is the slice width ϕ_w in arcsec;

`slice_height` is the slice height ϕ_h in arcsec;

`slit_width` is the slit width Φ_w in arcsec;

`slit_height` is the slit height Φ_h in arcsec;

`n_slices` is the number of slices;

`sig_ch2`, `sig_ch1`,`sig_ch3`, are the groove density of the grating in grooves mm^{-1} in Channel1, Channel2, and Channel3, respectively

`alp` is angle of incidence α in degrees;

`fcoll` is the collimator focal length f_{coll} in mm;

`fno_coll` is the collimator focal number fno_{coll};

`fcam` is the camera focal length f_{cam} in mm;

`fno_cam` is the camera focal number fno_{cam};

`m` is the diffraction order;

`wlc_ch1, wlc_ch2, wlc_ch3` are the central wavelength in Å for Channel1, Channel2, Channel3 respectively;

`pixsize` is the pixel size in μm;

`NXpix` number of detector pixels in the dispersion direction (X);

`NYpix` number of detector pixels in the dispersion direction (Y);

`d_tel` is the telescope diameter in mm;

`eps` is the telescope obstruction ϵ;

4 Formulae

The following Section gives an overview of all the formulae used by the ETC script to compute SNR. Atmosphere, seeing, telescope, CUBES spectrograph and detector effects are taken into account. At a given wavelength wl, SNR is computed for a compact astronomical object according to the main equation 1.

$$SNR_{wl} = \frac{PE^{obj} \cdot ExpT}{\sqrt{[PE^{obj} + PE^{sky}] \cdot ExpT + {N_{DC}}^2 \cdot ExpT + {N_R}^2}} \tag{1}$$

where:

PE^{obj} is the number of photo-electrons per second from the object, per wavelength bin (i.e. Å per pixel);

PE^{sky} is the number of photo-electrons per second from the sky, per wavelength bin;

${N_{DC}}^2$ is the detector noise due to the dark current in photo-electrons per second and integrated by summing the counts on the CCD area defined by the integer number of pixels in the dispersion direction (nx=1) corresponding to the wavelength bin and by the number of pixels in the spatial direction (ny) depending on the observational conditions (see below);

${N_R}^2$ is the detector read-out noise in photo-electrons per $nx \cdot ny$ pixels;

$ExpT$ is the integration time in seconds.

SNR is the Signal-to-Noise ratio.

By inverting equation 1, ExpT can be computed for a given SNR$_{wl}$.

4.1 Selection of the Channel

In order to select the Channel covering the user select wavelength, wl, the covered spectral range in each Channel is computed by using:

$$\lambda_{1,2} = wl_c \mp \frac{NXpix}{2} \cdot AMPIX \tag{2}$$

$$\tag{3}$$

where wl_c is the central wavelength of the each channel, NXpix is the number of detector pixels in the dispersion direction (X), AMPIX is the linear dispersion in Å/pix (see Section 4.7), and λ_1 and λ_2 are the starting and ending wavelength, respectively.

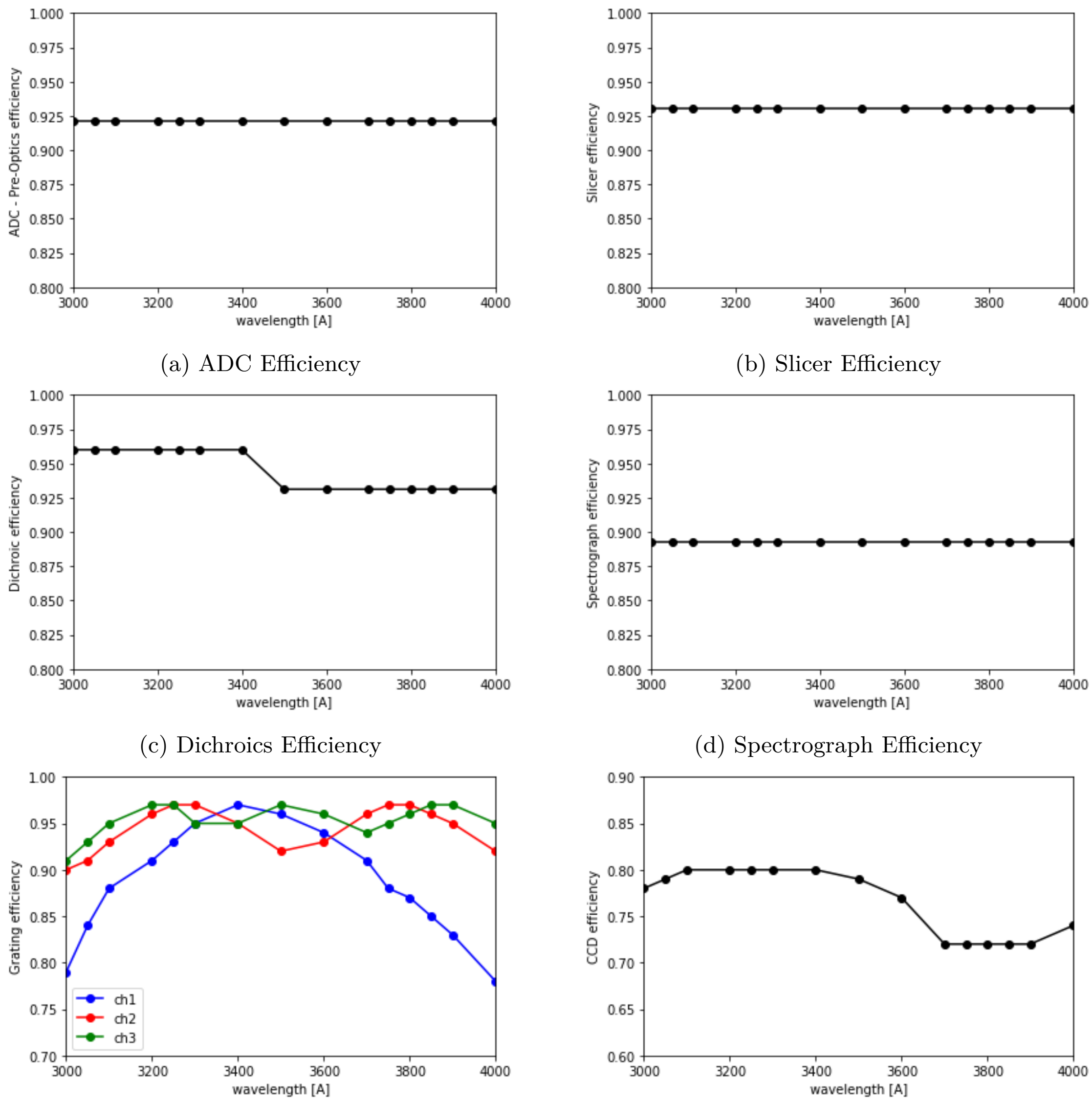
If the user select wavelength, wl, is lower than the λ_1 starting wavelength of the first grating or higher than the λ_2 ending wavelength of the third grating, a ”Warning” will be displayed in the output-window to alert the user that the line falls off the CCD.

4.2 Instrument Efficiency

The instrument efficiency is computed by taking into account the efficiency of all the optical components from the fore-optics to the detector. For each optical component an ASCII table of two columns (wavelength and throughput) must be provided by the responsible for the optical design. For each optical component the associated efficiency file is read and the value at user required wavelength, wl, is computed by a cubic spline interpolation. By applying this procedure to the ADC, Image Slicer, Dichroics, optics of the Spectrograph, Grating and CCD, the total instrument efficiency of the system QE_{IN} is given by:

$$QE_{\text{IN}} = QE_{\text{ADC}} \cdot QE_{\text{SLC}} \cdot QE_{\text{DCHR}} \cdot QE_{\text{SPECT}} \cdot QE_{\text{GRT}} \cdot QE_{\text{CCD}} \tag{4}$$

Figure 1 shows the temporary files adopted in the ETC for the different components that have to be updated with the foreseen efficiency.



(a) ADC Efficiency (b) Slicer Efficiency

(c) Dichroics Efficiency (d) Spectrograph Efficiency

(e) Grating Efficiency (f) CCD Efficiency

Figure 1: Efficiency of the different components. Updated in July 2020.

4.3 Telescope efficiency

A temporary file for the telescope efficiency of VLT is read and the value at the user required wl is computed through a cubic spline interpolation. The telescope throughput must take into account the reflectivity efficiency of the telescope mirrors and the normalized telescope obstruction (ϵ):

$$\tau_{tel} = (1 - \epsilon^2) \cdot Refl_{tel} \tag{5}$$

4.4 Overall instrumental DQE

Eventually, the overall detective quantum efficiency, DQE (slit losses are not yet included):

$$DQE = \tau_{tel} \cdot QE_{IN} \tag{6}$$

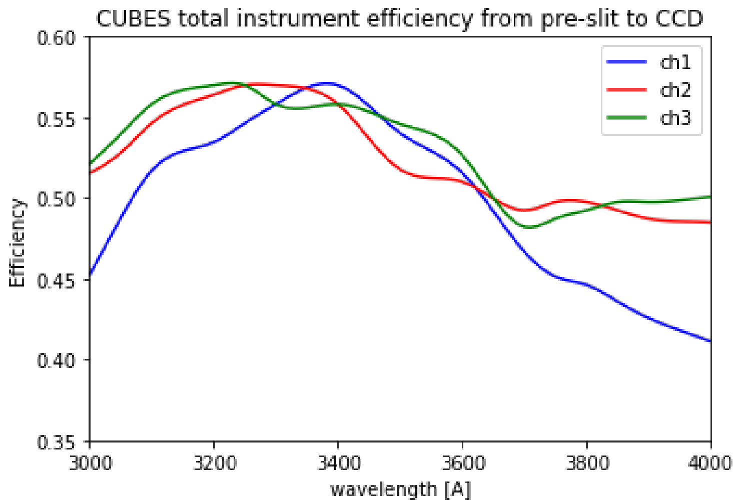


Figure 3: Total instrument efficiency from the pre-slit optics to the detector for the three channels.

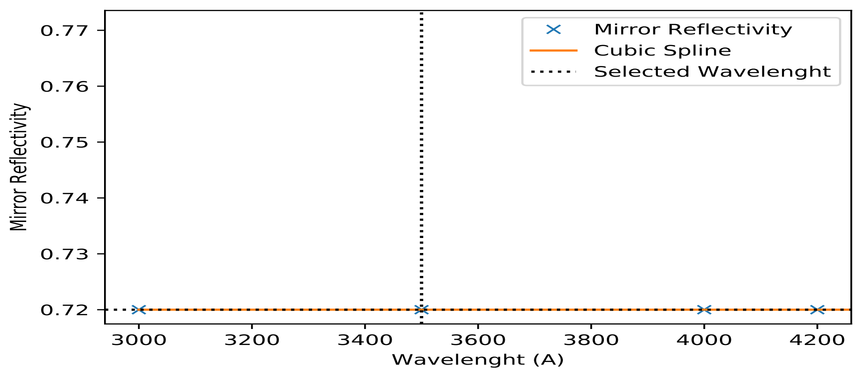


Figure 4: Telescope efficiency

For example, on the bases of these temporary files, at $wl = 3500\,\text{Å}$, the overall DQE from telescope to the CCD not including atmosphere and slit losses is equal to 0.4033

4.5 Atmospheric Transmission

A typical atmospheric transmission spectrum at the Paranal VLT observatory for unit air mass (X=1) and star at the zenith is read. If necessary, the atmospheric transmission at the user's required input airmass X is computed:

$$T_X(\lambda) = e^{\ln(T_{X=1})\cdot X} \tag{7}$$

Then, the code computes the transmission at wl through a cubic spline interpolation (see Figure 5).
For the current Phase A, the atmospheric transmission spectrum was generated by simply using the yearly average conditions at Paranal and at the zenith, by using http://www.eso.org/observing/etc/bin/simu/skycalc ESO code.

4.6 Total Efficiency

The overall transmittance of the system, DQE_{tot}, from the telescope to the detector, including atmosphere transmission and slit losses (i.e. percentage of the monochromatic image at the entrance slit that may be

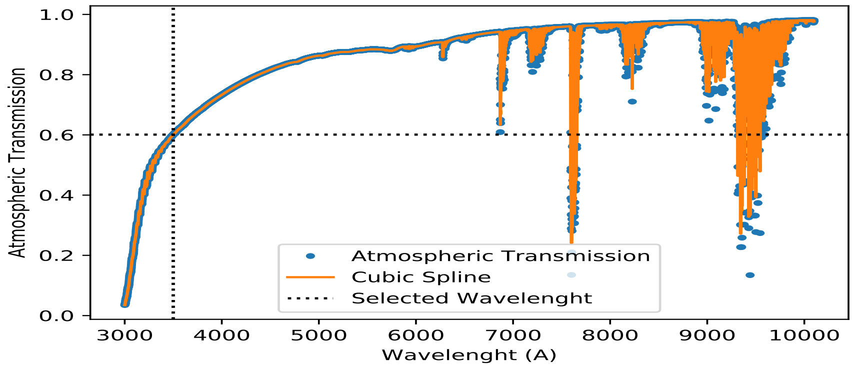


Figure 5: Atmospheric transmission spectrum.

intercepted by the slit jaws and not reach the detector) is given by:

$$DQE_{\mathrm{tot}} = T_{\mathrm{X}} \cdot \tau_{tel} \cdot QE_{\mathrm{IN}} \cdot k \tag{8}$$

where k takes into account factors not included in the transmittance of the system like slit losses.

Among the input parameters given by the user (see Table 1) there is the seeing Γ_0 which is given in arcsec at the wavelength $\lambda_0 = 5500$ Å. The seeing varies with the wavelength according to the following equation:

$$\Gamma(\lambda) = \Gamma_0 \cdot \left(\frac{\lambda}{\lambda_0}\right)^{-0.2} \tag{9}$$

In equation 9 no dependence on the zenith distance is assumed. The seeing at the user select wavelength (Γ_{wl}) is an indicator of the monochromatic image dimension and the slit throughput can be estimated as the ratio between the entrance slit width projected on the sky (Φ_w=1.5") and the diameter of the image (Γ_{wl}). With this assumption, for a point source, the factor k is given by:

$$k = \frac{\Phi_w}{\Gamma_{wl}} \tag{10}$$

In the case where $\Phi_w \leq \Gamma_{\mathrm{wl}}$, $k = 1$.

4.7 Spectral bin DW_BIN – Dispersion Scale (Å/pixel)

The spectral bin DW_BIN is the wavelength range in Å that is used for the integration over wavelength and corresponding to one pixel (nx=1). For a grating with a groove density σ (gr/mm) and an angle of incidence α, the angular dispersion ANGD [rad/mm] and the linear dispersion AM [Å/mm], in the order m, are given by:

$$ANGD = \frac{d\beta}{d\lambda} = \frac{m \cdot \sigma}{\cos\beta} \tag{11}$$

$$AM = \frac{10^7}{ANGD \cdot f_{cam}} \tag{12}$$

$$AMPIX = \text{pixsize} \cdot 10^{-3} \cdot AM = DW\_BIN \tag{13}$$

β is the diffraction angle where for the wavelength wl, f_{cam} is the camera focal length in mm, pixsize is the pixel size in μm, and from the grating equation :

$$\sin\beta = m \cdot wl \cdot 10^{-7} \cdot \sigma - \sin\alpha \tag{14}$$

$$\cos\beta = \sqrt{\left(1 - \sin^2\beta\right)} \tag{15}$$

and WL_BIN is the wavelength interval in Å for one pixel.

If reading the CCD a binning greater than one is used, then:

$$DW\_BIN = \text{nbinx} \cdot AMPIX \tag{16}$$

4.8 Spatial Scale ("/pixel)

The spatial scale ["/pixel] in the dispersion direction (ARCPX) and spatial direction (ARCPY) are given by:

$$XMARC = r \cdot \frac{f_{cam} \cdot D_{tel} \cdot 10^3}{d_{coll} \cdot 206264.8062} \tag{17}$$

$$YMARC = \frac{f_{cam} \cdot D_{tel} \cdot 10^3}{d_{coll} \cdot 206264.8062} \tag{18}$$

$$ARCPX = \frac{\text{pixsize}}{XMARC} \tag{19}$$

$$ARCPY = \frac{\text{pixsize}}{YMARC} \tag{20}$$

where XMARC and YMARC are μm/arcsec, d_{coll} in the collimator diameter, and r is the anamorphic magnification:

$$r = \frac{\cos\alpha}{\cos\beta} \tag{21}$$

Table 3: Angular and linear Dispersion for the three channels

	λ_c Å	ARCPX "/pix	ARCPY "/pix	AM Å/mm	AMPIX Å/pixel
Channel1	3218	0.122	0.122	5.37	0.081
Channel2	3513	0.122	0.122	5.87	0.088
Channel3	3825	0.122	0.122	6.39	0.095

By using the formulae 11-20 and according to the actual proposed configurations for the three channels (grating angle of incidence α=32.8 deg, f_{coll}=3094 and focal ratio $fno_{coll} = 20.63$, and f_{cam}=464 and focal ratio fno_{cam}=3.09), Table 3 shows the angular and linear dispersion values for the central-wavelengths of each sub-band (λ_c=3218, 3513, 3825 Å for Channel1, Channel2, and Channel3, respectively). Note the if we assume a diameter of 8000 mm as in in the last table that circulated among us, ARCPX and ARCPY is 0.125 "/pix.

The code computes the number (ny) of pixels integrated in the spatial (Y) directions by the detector as:

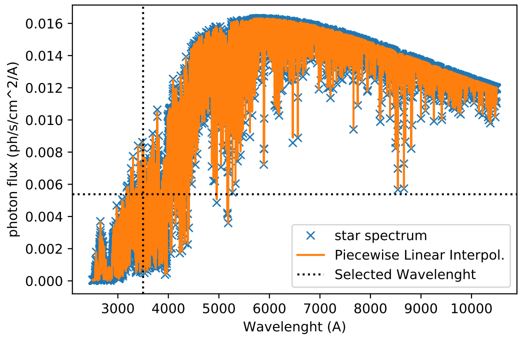
$$n_y = 6 \cdot \frac{\Gamma_{wl}}{ARCPY} \tag{22}$$

where 6 is the number of slitlets, Γ_{wl} is the seeing. So, in this preliminary version of the code, n_y is computed as 6 times the nearest integer value of the ratio between the seeing and the pixel size in arcsec.

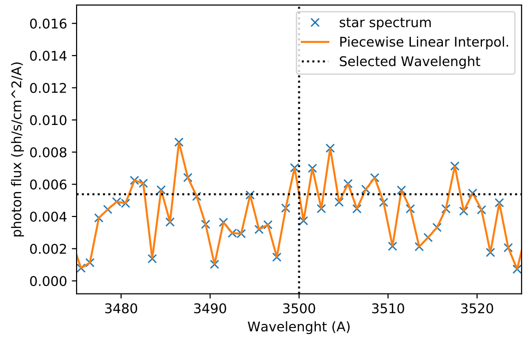
4.9 Target Spectrum

A target spectrum of a given visual magnitude, V_0, is read from an ASCII file as erg cm^{-2} s^{-1} Å^{-1} outside the atmosphere, converted in photon cm^{-2} s^{-1} Å^{-1}, scaled to the user required magnitude V

$$N^V_{obj}(\lambda) = N^{V_0}_{obj}(\lambda) \cdot 10^{-\frac{(V-V_0)}{2.5}} \tag{23}$$



(a) Star Spectrum



(b) Detail of Star Spectrum

and then the N_{wl}^{obj} flux at the user selected wavelength (wl) is computed through a piece-wise linear interpolation.
The target spectrum is an ASCII file with two columns (i.e. wavelength in Å and flux in $\mathrm{erg\,cm^{-2}\,s^{-1}\ \text{Å}^{-1}}$). In this preliminary version the target spectrum may be selected by the user via the ETC input interface and it may be:

- a **synthetic spectrum** out of some synthetic spectra extracted from the library by Munari et al. 2005 (A&A 442, 1127) at a resolution of 1 Å[1];

- a **quasar spectrum** from Zheng et al. 1997 (ApJ 475, 469), actually a composite of 101 quasar spectra observed with HST FOS and redshifted at $z = 2$.

- a **custom spectrum** to upload. An ASCII file with two numerical columns i.e. wavelength in Å and flux in unit proportional to $\mathrm{erg\,cm^{-2}\,s^{-1}\ \text{Å}^{-1}}$. The spectrum will be scaled to the V magnitude. Spectrum (red-shifted is applied) must cover the spectral range of the instrument mode but also of the photometric V band.

The magnitude in AB units of the object is computed at the user selected wl wavelength, according to the equation:

$$m_{obj}(AB) = -2.5\log\Big(\frac{N_{wl}}{1\,\mathrm{photon\,cm^{-1}\,s^{-1}\,\text{Å}^{-1}}}\Big) - 2,5\log\Big(\frac{wl}{1\,\text{Å}^{-1}}\Big) + 16.85 \tag{24}$$

4.10 Sky Background

Likewise the star spectrum, a sky background spectrum $\mathrm{photon\,cm^{-2}\,s^{-1}\ \text{Å}^{-1}\,arcsec^{-2}}$ at given magnitude V_0 is read and scaled to the user's required V_{sky} magnitude according to:

$$N_{sky}^{V}(\lambda) = N_{sky}^{V_0}(\lambda)\cdot 10^{-\frac{(V-V_0)}{2.5}} \tag{25}$$

Then, the value of flux N_{wl}^{sky} at wl user value is extracted through a cubic spline interpolation.

The sky magnitude in AB units is also computed as:

$$m_{sky}(AB) = -2.5\log\Big(\frac{N_{wl}^{sky}}{1\,\mathrm{photon\,cm^{-1}\,s^{-1}\,\text{Å}^{-1}\,arcsec^{-2}}}\Big) - 2,5\log\Big(\frac{wl}{1\,\text{Å}^{-1}}\Big) + 16.85 \tag{26}$$

[1]These example spectra were downloaded from http://wwwuser.oats.inaf.it/castelli/spectra.html and interested people may download from this site other spectra that can use as "Custom spectrum". They are in the correct format but you only need to eliminate the header comment lines.

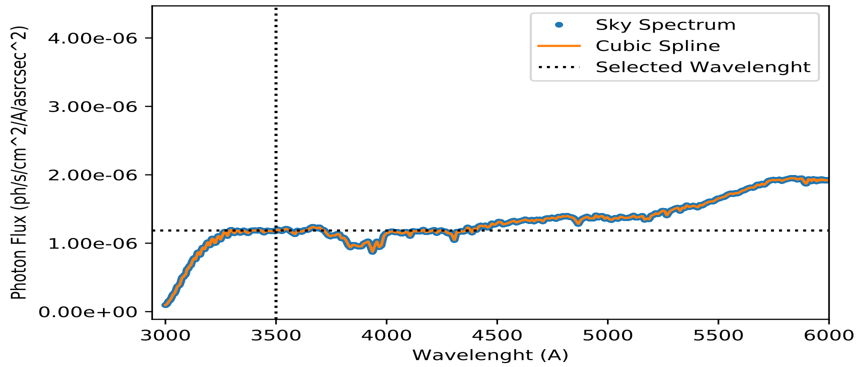


Figure 7: Sky background spectrum. Continuum background in between airglow sky-lines. Generated by using skycalc ESO code, in dark condition: U=22.22, B=22.87, and V=22.15 mag/arcsec^2.

4.11 Photon Fluxes: PE^{obj} and PE^{sky}

The photon flux (phot-el/sec) collected by the telescope and transmitted to the detector for the object and the sky are then computed as follows:

$$PE^{obj} = N^{obj}_{wl} \cdot \frac{\pi}{4} \cdot \left(\frac{D_{tel}}{10}\right)^2 \cdot T_{\mathrm{X}} \cdot \tau_{tel} \cdot QE_{\mathrm{IN}} \cdot k \cdot \Delta\lambda \tag{27}$$

$$PE^{sky} = N^{sky}_{wl} \cdot \frac{\pi}{4} \cdot \left(\frac{D_{tel}}{10}\right)^2 \cdot T_{\mathrm{X}} \cdot \tau_{tel} \cdot QE_{\mathrm{IN}} \cdot \Delta\lambda \cdot \Omega \tag{28}$$

where:

D_{tel} is the telescope diameter in mm;

Ω is the detector area in arc-seconds square projected on the sky as the sky is an extended source: $\Omega = \Phi_w \cdot ny \cdot ARCPY$ arcsec^2.

$\Delta\lambda$ is the wavelength bin WL_BIN.

The number of photons collected by the detector over the exposure time is given by:

$$\tilde{N}^{obj} = PE^{obj} \cdot ExpT \tag{29}$$

$$\tilde{N}^{sky} = PE^{sky} \cdot ExpT \tag{30}$$

4.12 Detector Noise

Detector noise is computed as:

$$NOISE_{DET} = \sqrt{N^2_{DC} \cdot ExpT + N^2_R} \tag{31}$$

where:

$$N^2_{DC} = n_x \cdot n_y \cdot DC \tag{32}$$

$$N^2_R = \frac{n_x\, n_y}{bin_x\, bin_y} \cdot RON^2 \tag{33}$$

where:

N^2_{DC} is the detector noise due to the dark current in photo-electrons per second:

N^2_R is the detector read-out noise in photo-electrons integrated on $nx \cdot ny$ pixels;

T_0 is reference temperature (in $°C$) for the dark current

T_{det} is the working temperature (in $°C$) of the detector

DC is dark current in e$^-$/sec/pixel at T_0

bin_x is binning in x direction

bin_y is binning in y direction

RON is read out noise in e$^-$/pix

Total noise can finally be computed!

$$NOISE = \sqrt{\tilde{N}^{obj} + \tilde{N}^{sky} + NOISE^2_{DET}} \tag{34}$$

4.13 SNR from given Exposure Time

Finally, SNR will be evaluated as:

$$SNR = \frac{\tilde{N}^{obj}}{NOISE} \tag{35}$$

The procedure computes SNR for different exposure times in order to create Figure 8 showing SNR for different exposure time in the case of the specific selected target and observing conditions:

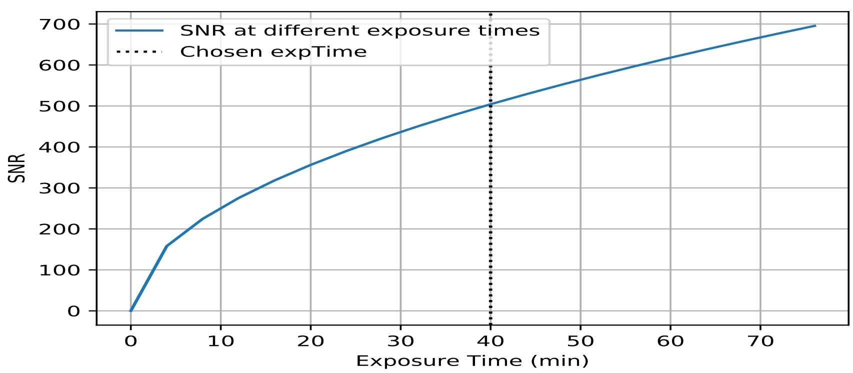


Figure 8: SNR at different Exposure Times

4.14 Exposure Time from given SNR

A prediction of the Exposure Time can be made for a desired SNR by inverting Formula 1 in order to extract ExpT.

$$ExpT = \frac{(PE^{obj} + PE^{sky} + N^2_{DC}) \cdot SNR^2 + \sqrt{[(PE^{obj} + PE^{sky} + N^2_{DC}) \cdot SNR^2] \cdot 2 + 4\,(PE^{obj})^2 \cdot N^2_R \cdot SNR^2}}{2\,(PE^{obj})^2} \tag{36}$$

Also in this case a plot like that shown in Figure 8 is computed and displayed via a graphical interface.

4.15 Magnitute Limit

A magnitude limit in AB unit is computed at wl, SNR=3 and an exposure time (ExpT) given by the user in input. SNR has been adopted as a typical minimum value necessary for astrophysical purposes. This magnitude is displayed to the user together with the plot shown in Figure 1.

Moreover, the magnitude limit for different SNR at a given exposure time may also be computed. By using equations 29, 30 and 31 the script will compute $\tilde{N}^{sky}$, $\tilde{N}^{obj}$ when the AB magnitude is zero and $NOISE_{DET}$ for a selected exposure time. The noise due to sky and detector is given by :

$$NOISE = \sqrt{\tilde{N}^{sky} + NOISE^2_{DET}} \tag{37}$$

$$ABMAG_{lim} = -2.5 \log \left[\frac{SNR^2}{2\ \tilde{N}^{obj}} \cdot \left(1 + \sqrt{1 + 4\ \frac{NOISE^2}{SNR^2}} \right) \right] \tag{38}$$

This procedure will be iterated over different values of SNR, resulting in the following plot:

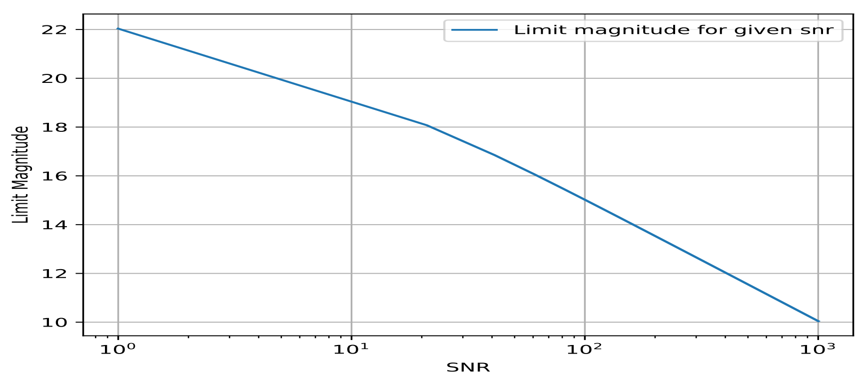


Figure 9: AB magnitude limit versus SNR diagram computed for Expt=600 sec, wl = 3500 Å, synthetic spectrum of $T_{\rm eff}$=5250 K, $\log g$=4.5, [Fe/H]=0, and V_{sky}=22.15 mag/arcsec^2.